বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

Question Paper

B.Sc. Honours Examination 2021

(Under CBCS Pattern)

Semester - VI

Subject: MATHEMATICS

Paper: DSE 3-T

Full Marks : 60

Time : 3 Hours

Candiates are required to give their answer in their own words as far as practicable.
The figures in the margin indicate full marks.

MECHANICS

Answer any ***four*** questions from the following. 4×15=60

1. (a) A circular disc, of radius a, whose density is proportional to the distance from the centre, has a hole cut in it bounded by a circle of diameter b, which passes through the centre. Show that the distance of the centre of gravity of the remaining portion from the centre of the disc is $\dfrac{6b^4}{15\pi a^3 - 10b^3}$.

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(b) A solid frustum of paraboloid of revolution of height 'h' and latus rectum '$4a$', rests, with its vertex on the vertex on the vertex of a paraboloid of revolution, whose latus rectum is '$4b$'. Show that the equilibrium is stable if $h < \dfrac{3ab}{a+b}$. 8+7

2. (a) A perfectly rough plane is inclined at an angle α to the horizon; show that the least eccentricity of the ellipse which can rest on the plane is $\sqrt{\dfrac{2\sin\alpha}{1+\sin\alpha}}$.

(b) Define astatic equilibrium A system of coplanar forces is equivalent to a couple of moment M and if the forces are turned through a right angle about their respective points of application in the same sense, they are equivalent to a couple *N*. Prove that when each forces be turned about its point of application through an angle α in the same sense, the system will be in equilibrium if $\tan\alpha = -\dfrac{M}{N}$. Also show that if the forces be turned through the angle $2\tan^{-1}\left(\dfrac{N}{M}\right)$, they would still be equivalent to a couple of moment *M*. 7+8

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3. (a) Two equal forces act along the generators of the same system of the hyperboloid $\dfrac{x^2+y^2}{b^2} = \dfrac{z^2}{a^2} = 1$ and cut the plane $z = 0$ at the extremities of perpendicular diameters of the circle $x^2 + y^2 = b^2$. Show that the pitch of the equivalent wrench is $\dfrac{ab^2}{2a^2+b^2}$.

(b) A solid hemisphere is supported by a string fixed to a point on the rim and to a point on a smooth vertical wall which the curved surface of the hemisphere is in contact. The inclinations of the string and the plane base of the hemisphere to the vertical are θ and ϕ respectively. Using the principle of virtual work, prove that $\tan\phi = \dfrac{3}{8} + \tan\theta$. 7+8

4. (a) A particle is projected from the origin with a velocity whose components along the axes are $\left(0, -\dfrac{a}{p}, V\right)$ under a force per unit mass whose components along the axes are $(a\cos pt,\ 0,\ a\sin pt)$. Find the path of the particle.

(b) Assuming that the moon is acted upon by a force $\frac{\mu}{(\text{distance})^2}$ to the earch and effect of the Sun's disturbing force is to cause a force m^2 × disctance from the earth to the moon. Show that, the orbit being nearly circular, the apsidal angle is nearly $\mu\left\{1+\frac{3m^2}{2n^2}\right\}$ where the lunar month is $\frac{2\pi}{n}$ and cubes of *m*are neglected. 8+7

5. (a) A particle is projected with velocity u at an inclination α above the horizontal in a medium whose resistance per unit mass is k times the velocity. Show that its direction will again make an angle α below the horizontal after a time $\frac{1}{k}\log\left(1+\frac{2ku}{g}\sin\alpha\right)$.

(b) A planet, of mass M and periodic time T, when at its greatest distance from the Sun comes into a collision with a metereor of mass m, moving in the same orbit in the opposite direction with velocity v. If $\frac{m}{M}$ be small, find the change of length of major axis. 8+7

6. (a) Show that the moment of inertia of a homogeneous triangular lamina ABC about a line through A and perpenducular to the plane of the triangle is $\frac{m}{12}\left(3b^2+3c^2-a^2\right)$ where a, b, c are the lengths of sides BC, CA and AB and M is mass of the triangle lamina.

(b) A non homogeneous rod AB, of length $2l$, whose density at any point is directly proportional to the distance of the point from A, is rotating with a uniform angular velocity ω about a vertical axis through A. If the rod is inclined at an angle α to the vertical, show that the value of α is either 0 or $\cos^{-1}\left(\frac{2g}{2l\omega^2}\right)$. 8+7

7. (a) Show that the rate of changes of the angular momentum of a rigid body about the axis of rotation is equal to the sum of moments about the same axis of all forces acting on the body.

(b) Three equal uniform rods placed in a straight line are freely joined at junctions and move with velocity v perpendicular to their lengths. If the middle point of the middle

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rod be suddenly fixed, show that the ends of other two rods will meet in time $\frac{4\pi a}{9v}$ where 'a' is the length of each rod.

(c) A solid homogeneous cone, of height h and vertical angle 2α, oscillates about a horizontal axis through its vertex. Show that the length of the simple equivalent pendulum is $\frac{h}{5}\left(4+\tan^2\alpha\right)$. 5+5+5

8. (a) A rough uniform rod, of length $2a$, is placed on a rough table at right angles to its edge. If the centre of gravity of the rod be initially at a distance b beyond the edge, then show that the rod will begin to slide when it has turned through an angle $\tan\theta = \frac{\mu a^2}{a^2+9b^2}$, where μ is the coefficient of friction.

(b) Three unequal uniform rods AB, BC, CD are freely jointed and placed in a straight line on a smooth table. The rod AB is struck at its end A by a blow which is perpendicular to its length. Find the resulting motion and show that the velocity of the centre of AB is 19 times that of CD and its angular velocity 11 times that of CD.

7+8

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Or

NUMBER THEORY

Answer any ***four*** questions from the following. $4\times15=60$

1. (a) Prove that, for a positive integer n and any integer a, $\gcd(a,\ a+n)$ divides n; hence deduce that $\gcd(a,\ a+1)=1$. 3

   (b) Determine all solutions in the positive integers of the Diophantine edquation:

   $54x+21y=906$ 2

   (c) Show that the Goldbach conjecture implies that for each even integer $2n$ there exist integers n_1 and n_2 with $\sigma(n_1)+\sigma(n_2)=2n$. 3

   (d) If $p\geq q\geq 5$ and p, q are both primes, prove that $24 \mid p^2-q^2$. 4

   (e) If p is a prime and $k\geq 2$ then show that $\phi(\phi(p^k))=p^{k-2}\phi((p-1)^2)$. 3

2. (a) Show that there are infinite numbers of primes of the form $4k+3$. 2

   (b) Find the values of $n\geq 1$ for which $n!.(n+1)!.(n+2)!$ are a perfect square. 2

   (c) Find the remainder when $\sum_{k=1}^{200} k!$ is divided by 8. 3

   (d) Prove that 53^{100} + 103^{53} is divisible by 39. 5

   (e) Show that the product of any set of n consecutive integers s divisible by n. 3

3. (a) State and prove Chinese Remainder Theorem. 5

   (b) Find the smallest integer $a>2$ such that $2|a,\ 3|a+1,\ 4|a+2,\ 5|a+3,\ 6|a+4$. 3

   (c) If 7 does not divide a then show that either a^3+1 or a^3-1 is divisible by 7. 3

   (d) Find the remainder when 2222^{5555} + 5555^{2222} is divided by 7. 4

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4. (a) If p is a prime then show that $(p+1)!+1 = 0(\bmod p)$. 5

(b) Show that $4(29!)+5! \equiv 0 \,(\bmod 31)$. 3

(c) Show that the function τ and σ are multiplicative functions. 4

(d) Prove that the number of primes is infinite. 3

5. (a) For each positive integer $n \geq 1$, show that $\sum_{d|n} \mu(d) = \begin{cases} 1 & \text{if} \quad n = 1 \\ 0 & \text{if} \quad n > 1 \end{cases}$ where d runs through the positive divisors of n and μ is the Möblius function. 5

(b) Show that $n! = \prod_{p \leq n} p^{\sum_{k=1}^{\infty}\left[\frac{n}{p^k}\right]}$. 8

(c) If x is not an integer then find the value of $[x]+[-x]$. 2

6. (a) If $p \neq 5$ is an odd prime, prove that either $p^2 - 1$ or $p^2 + 1$ is divisible by 10. 3

(b) If $n \geq 1$ and $\gcd(a, n) = 1$ then show that $a^{\phi(n)} \equiv 1(\bmod n)$. Deduce the Fermat's theorem from it. 6

(c) $\gcd(a, n) = 1$ and $\gcd(a-1, n) = 1$ then show that $1 + a + a^2 + a^3 + \ldots + a^{\phi(n)-1}$ is divisible by n. 3

(d) For $n > 1$, show that the sum of the positive integers less than n and relatively prime to n is $\frac{1}{2} n\phi(n)$. 3

7. (a) If n has a primitive root then show that it has exactly $\phi(\phi(n))$ numbers of primitive roots. 2

(b) If $\gcd(m, n) = 1$ then show that the integer mn has no primitive root. $(m > 2,\ n > 2)$. 2

(c) Solve the quadric congruence : $5x^2 + 6x + 1 \equiv 0 \,(\bmod 23)$. 5

(d) If p, q are distinct odd prime then show that $(p/q)(q/p) = (-1)^{\left(\frac{p-1}{2}\right) \cdot \left(\frac{q-1}{2}\right)}$. 6

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8. (a) The ciphertext B ZRVU GRNPEKSTWSQ has been enciphered with the cipher $C_1 \equiv 3P_1 + 5P_2 \pmod{26}$, $C_2 \equiv 8P_1 + 13P_2 \pmod{26}$. Derive the plaintext. 5

(b) Define primitive Pythagorean triple. If x, y, z is a primitive Pythagorean triple, then prove that one of the integers x or y is even, while the other is odd. 3

(c) Show that the equation $x^4 + y^4 = z^4$ has no solution in the positive integers. 7

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Or

INDUSTRIAL MATHEMATICS

Answer any ***four*** questions from the following. $4\times15=60$

1. What is Medical Imaging ? Provide a framework of Inverse Problem.

2. Discuss Geological anomalies in Earth's interior from measurements at its surface.

3. Explain Beers Law with a suitable illustration.

4. Write short note on X-ray Behaviour.

5. Define Random Transform with an example.

6. Mention various properties of Back Projection.

7. Discuss the Algorithms of CT Scan Machine.

8. Mention the properties of Inverse Fourier Transforms in CT Scan.

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