বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

Question Paper

B.Sc. Honours Examinations 2021

(Under CBCS Pattern)

Semester - II

Subject : MATHEMATICS

Paper : GE 2-T

Algebra

Full Marks : 60

Time : 3 Hours

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

Answer any ***four*** of the following questions : $4 \times 15 = 60$

1. (a) Show that the roots of $(1 + z)^n = (1 - z)^n$ are the values of $i\tan\frac{r\pi}{n},\ r = 1, 2, \ldots\ldots, (n-1)$ but omitting $\frac{n}{2}$ if n is even. 6

   (b) Find the value of the sum of the 49th power of the roots of the equation $x^{13} - 1 = 0$. 3

   (c) The roots of the equation $x^3 + px^2 + qx + r = 0\ (r \neq 0)$ are α, β, γ. Find the equation whose roots are $\frac{1}{\alpha}+\frac{1}{\beta}-\frac{1}{\gamma}, \frac{1}{\beta}+\frac{1}{\gamma}-\frac{1}{\alpha}, \frac{1}{\gamma}+\frac{1}{\alpha}-\frac{1}{\beta}$. 6

2. (a) Solve the equation by Ferrari's Method : $x^4 - 6x^2 + 16x - 15 = 0$ 6

(b) Let A and G are arithmetic mean and geometric mean respectively of n positive real numbers $a_1, a_2, \ldots\ldots, a_n$.

Prove that $(1+A)^n \geq (1+a_1)(1+a_2)\ldots\ldots(1+a_n) \geq (1+G)^n$ 6

(c) Prove that the value of the product of first n odd positive integers is less than n^n. 3

3. (a) Consider a set Z in which the relation ρ is defined by aρb iff 2a + 3b is divisible by 5. Examine whether ρ is an equivalence relation. 6

(b) If $f : A \to B$ and $g : B \to C$ are both injective mapping then show that the composite mapping $gof : A \to C$ is injective. 3

(c) If $\left(2+\sqrt{3}\right)^n = I + f$ where I and n are positive integers and $0 < f < 1$, show that I is an odd integer and $(1 - f)(I + f) = 1$. 6

4. (a) Find the remainder when $1^5 + 2^5 + \ldots\ldots + 80^5$ is divided by 4. 3

(b) Find the integers u and v such that $20u + 63v = 1$. 3

(c) Show that a composite number has at least one prime divisor. 3

(d) If gcd $(a, b) = 1$, show that gcd $(a + b, a^2 + b^2 - ab) = 1$ or 3. 4

(e) Let A be a set of n elements and B be a set of m elements. Find the numbers of mapping from A to B. 2

5. (a) Determine the conditions for which the system : $x + y + z = 1$; $x + 2y - z = b$; $5x + 7y + az = b^2$ admits of (i) unique solution, (ii) no solution and (iii) many solution.s 8

(b) Prove that the rank of a real skew symmetric matrix cannot be 1. 3

(c) If A is a real orthogonal matrix and $I + A$ is non-singular, prove that the matrix $(I + A)^{-1} (I - A)$ is skew symmetric.

6. (a) Find the eigen values of the matrix $A = \begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{pmatrix}$. Find the eigen vectors corresponding to the eigen values with largest magnitude. 5

(b) Verify Cayley-Hamilton theorem for the matrix : $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$. Hence deduce A^{-1} if exists. 5

(c) Prove that a linearly independent set of vectors in a finite dimensional vector space V over a field F is either a basis of V or it can be extended to a basis of V. 5

7. (a) Determine the linear mapping $T : R^3 \rightarrow R^3$ that maps the basic vectors (0, 1, 1), (1, 0, 1), (1, 1, 0) of R^3 to the vector (2, 1, 1), (1, 2, 1), (1, 1, 2) respectively. Find Ker T and Img T. Shoe that $Dim\ Ker\ T + Dim\ Img\ T = 3$. 4

(b) Obtain fully reduced normal form of the matrix $A = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & 1 & 2 \end{pmatrix}$. Find non singular matrix P, Q such that PAQ is the fully reduced normal form. 4

(c) Let $T : R^2 \rightarrow R^3$ be defined by $T(x_1, x_2) = (x_1 - x_2, x_1, 2x_1 + x_2)$. Let β be the standard ordered basis of R^2 and $\gamma = \{(1,1,0), (0,1,1), (2,2,3)\}$. Compute $[T]_\beta^\gamma$. If $\alpha = \{(1,2), (2,3)\}$, Compute $[T]_\alpha^\gamma$. 4

(d) Let V and W be vector spaces and $T: V \rightarrow W$ be linear. Then show that $N(T)$ and $R(T)$_ are subspaces of V and W respectively. 3

8. (a) Do the polynomials $x^3 - 2x^2 + 1$, $4x^3 - x + 3$ and $3x - 2$ generate the vector space $P_3(R)$? Justify. 3

(b) Let $T : P(R) \rightarrow P(R)$ be defined by $T(f(x)) = f'(x)$. If T is linear, prove that T is onto but not one-to-one. 4

(c) If n be a positive integer and $(1 + 2i)^n = a + ib$, then prove that $a^2 + b^2 = 53^n$. Hence express 53^3 as a sum of two squares. 4

(d) If $bc = ac\ (mod\ n)$ and gcd $(c, n) = 1$, show that $b = a\ (mod\ n)$. 4